Impact, co-impact, cross-impact

A (short) review

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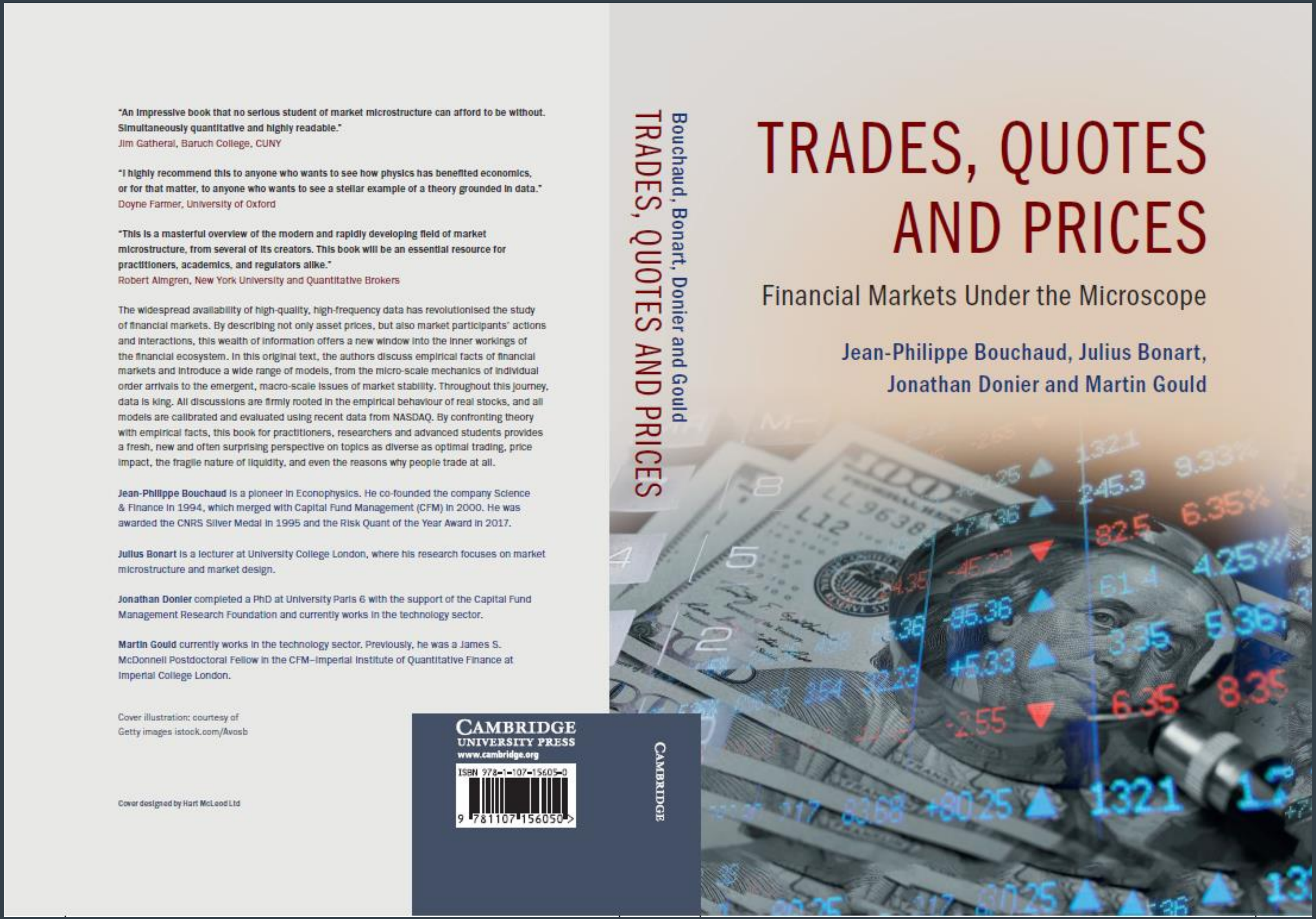
With thanks to M. Benzaquen, J. Bonart, F. Bucci, J. Donier, Z. Eisler, L. Garcia, F. Lillo, I. Mastromatteo, B. Toth

Disclaimer

Trades, Quotes and Prices



The basics: Impact & Transaction Costs (aka ‘slippage’)

You have a quantity Q >> V_{ask} to buy – say – what should you expect?

- Q has to be sliced and diced, and executed over some time T (~ days)
- During T, price will move as **$\Delta p = \pm \sigma\sqrt{T}$ + Impact(Q,T) [+ « alpha »]**
- The first term is execution **risk**: can be painful but vanishes on average
- The Impact term is usually **smaller** than $\sigma\sqrt{T}$ but **always adverse** → « slippage »
- The alpha term is usually **small** for T ~ days, but may become significant for fast signals

- **How large are these impact costs** (on top of other costs – fees, spreads, etc.)?
- How does impact depend on both Q and T?
- Many more interesting and relevant questions (see below)

Price impact: orders of magnitude

- Until the mid-nineties, the lore was that trading a quantity Q would impact prices as:

$\Delta p/p \sim Q/Mcap$ ~ **50 bp** for Q = 100% V (V = Average Daily Volume)

Note: Portfolio Insurance in 1987 should have moved the market by 0.1% (→ underestimates market fragility)

- Kyle (1985): theory for impact where an insider hides in the flux of noise traders

$\Delta p/p = \sigma N^{1/2}\ Q/V$ ~ **60 bp** for Q = 1% V ; σ = 2% ; N=1000 trades

Note: linear, permanent impact – assumes order flow to be uncorrelated

- Empirically: the 'square-root' law (1995 → 2020)

$\Delta p/p = Y\ \sigma\ (Q/V)^{1/2}$ ~ **10 bp** for Q = 1% V ; σ = 2% ; Y=0.5 (>> fees)

Note: non-linear, decaying impact – see below

Sqrt-Impact of Metaorders

A metaorder of size Q has a sqrt price midpoint impact:

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$

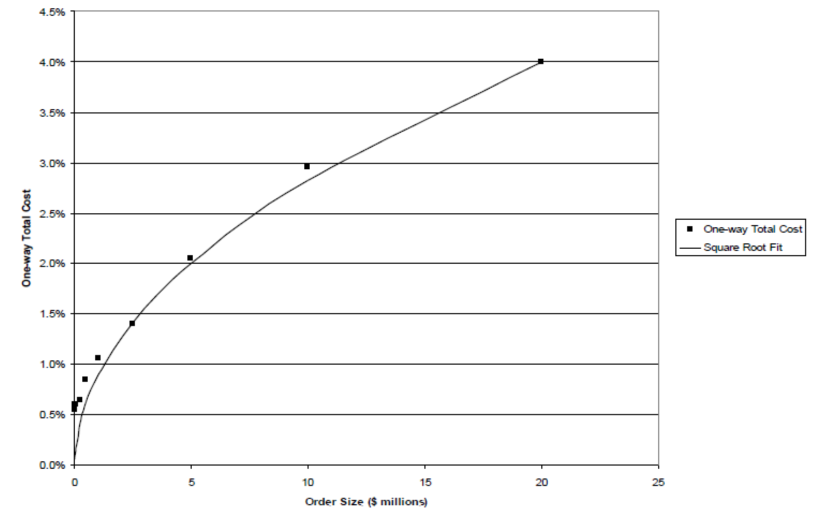
$I(Q)$ is the signed average price change
Q is the volume of the metaorder
σ_T is the volatility of the market
V_T is the total volume traded in the market
(Y of order 1)

Important notes:

- **Impact is usually small compared to vol itself (more later)**
- Requires a lot of averaging to be seen
- Beware of (many) conditioning artefacts
- Note: most data is in a « reasonable » trading regime (neither too big nor too fast, otherwise spreads might considerably increase)

A universal empirical result?

Independently but consistently reported by many groups since the mid-eighties (Loeb 83 (!), BARRA 95, Almgren 05, Engle, Kissel, JPM, DB, LH, CFM, Ancerno data, AQR)



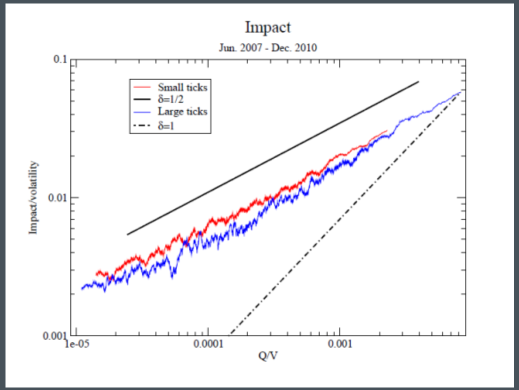
US Stocks, Loeb

Sqrt-Impact of Metaorders

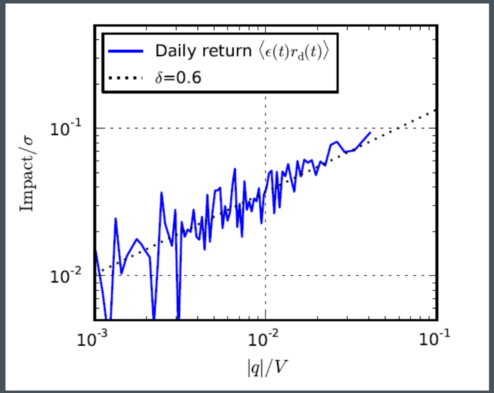
A universal empirical result? (CFM data)

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$

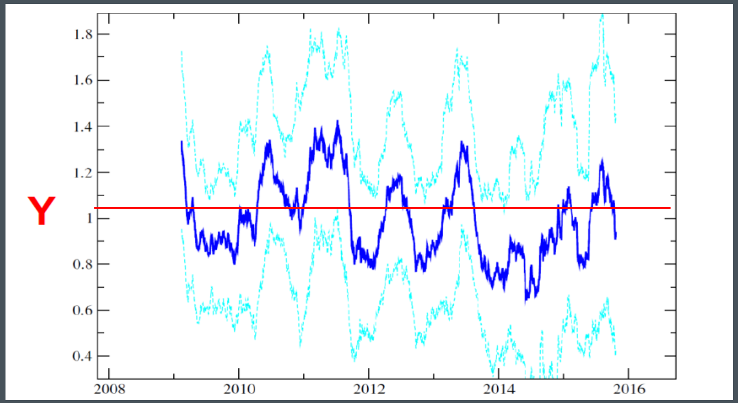
Futures



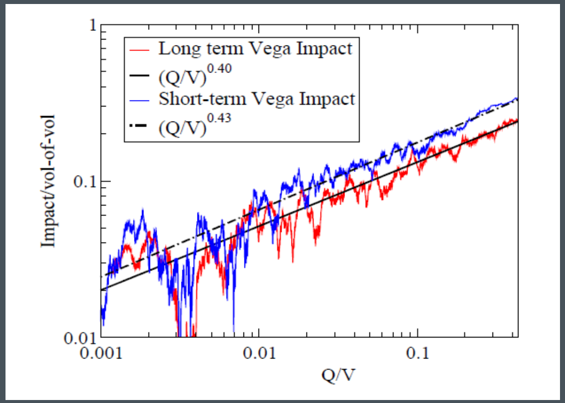
Intl stocks



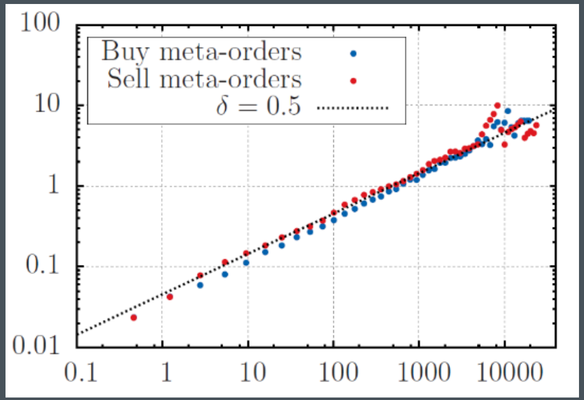
Remarkable stability of Y (+ noise)



US stock implied vol



Bitcoin! (J. Donier)

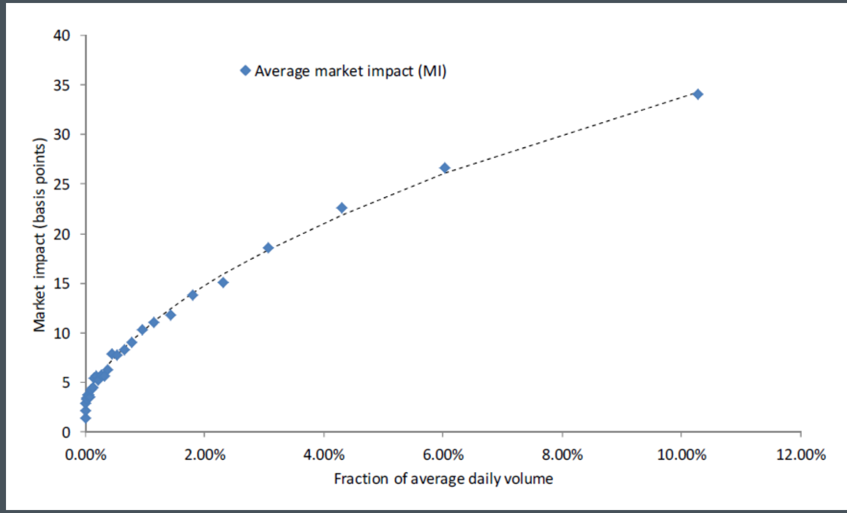


Sqrt-Impact of Metaorders

A universal empirical result? (AQR, Ancerno)

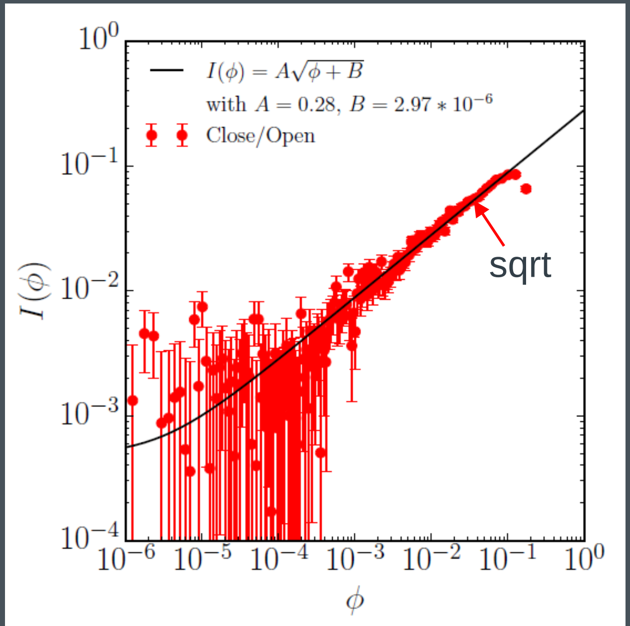
$$I(Q) = Y\sigma_T\sqrt{\frac{Q}{V_T}}$$

AQR: Intl stocks



Note: execution mostly with *limit* orders (Frazzini et al. 2018)

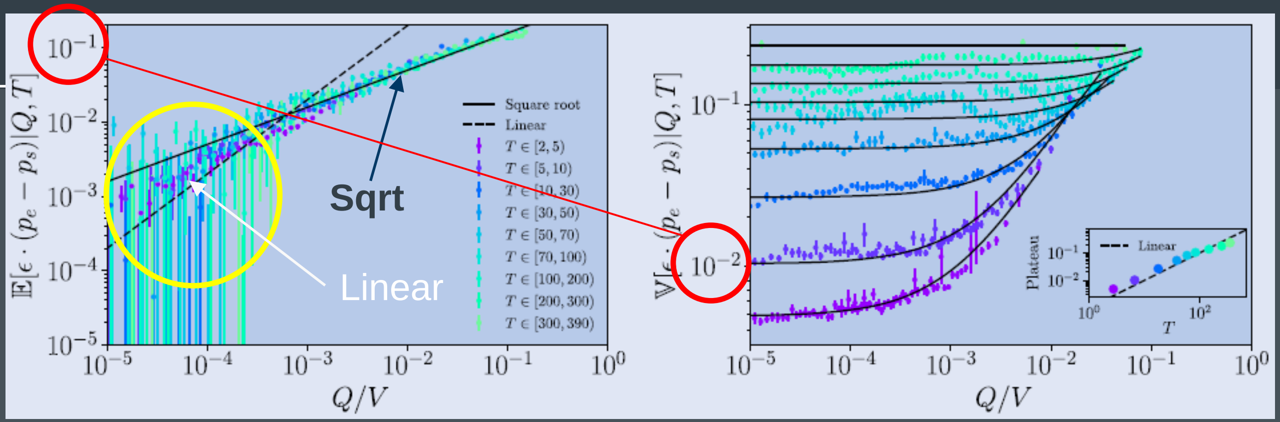
Ancerno: US 2007-2010



8M metaorders

The square-root impact law

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$



- Impact is very small compared to volatility
- Non linear: the second Q/2 impact less than the first Q/2!
- Impact is, to first approximation, independent of the execution time of the metaorder – but beware “spread impact”
- Remarkable stability of results: strategies, markets, execution, period (1980 – 2020), tick sizes, treatment of data,
- Hints that microstructure and HFT effects are not relevant, only ‘macro-liquidity’ (cf: same with limit orders, w/o HFT)
- Understanding its origin is important both conceptually and for applications
- Other relevant questions, e.g. how does this impact decay with time?

A dynamical theory of *latent* liquidity: the LLOB model (Donier, Bonart, Mastromatteo, JPB)

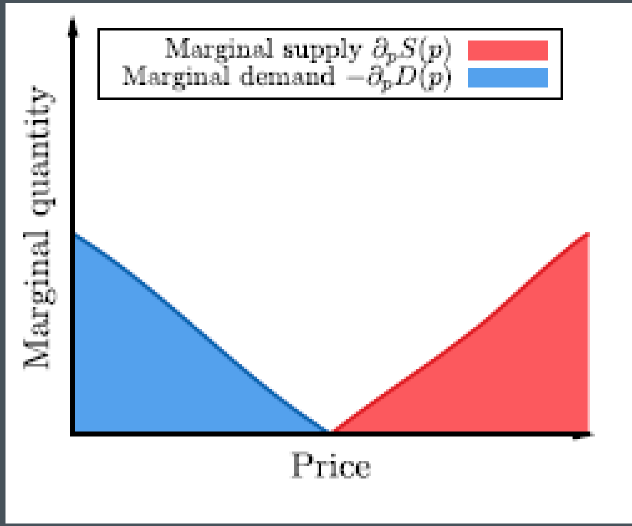
- A model for the "latent" order book, i.e. macro-liquidity (**not** micro/HFT liquidity!), inspired from a zero-intelligence ABM

$$\partial_t \varphi_b = D\partial_{xx}\varphi_b - \nu\varphi_b + \lambda\Theta(x_t - x) - R_{ab}(x)$$

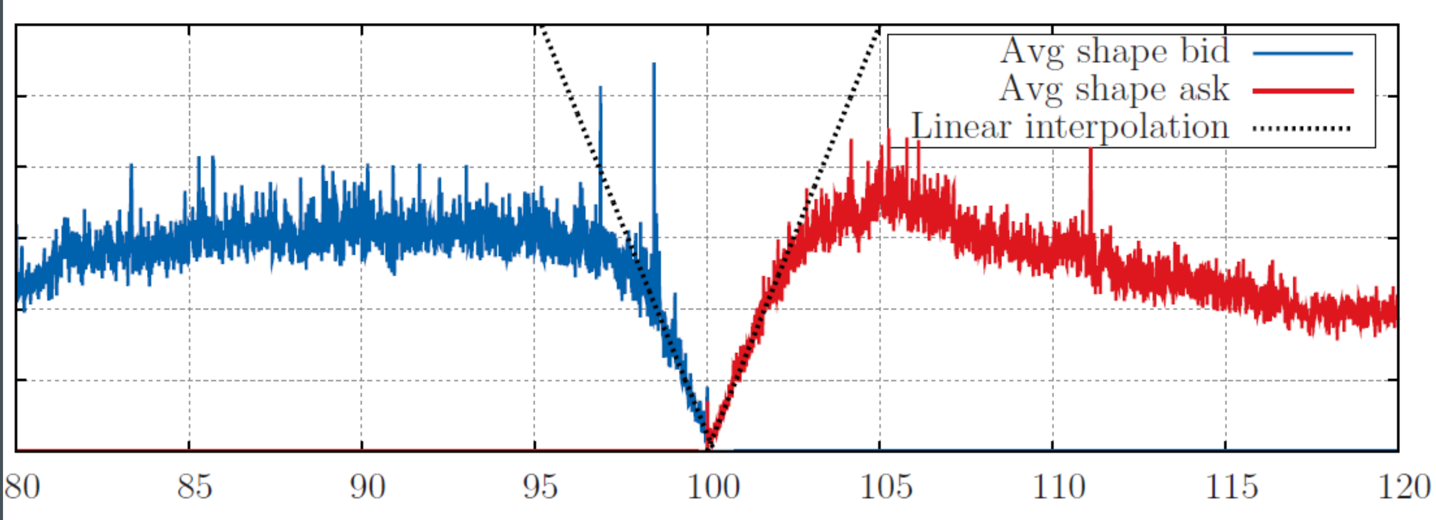
$$\partial_t \varphi_a = D\partial_{xx}\varphi_a - \nu\varphi_a + \lambda\Theta(x - x_t) - R_{ab}(x)$$

x_t:= current price

- Equilibrium state: a locally linear supply/demand curve – liquidity at the current price is at a **V-shaped minimum**



Theory



Bitcoin 2012

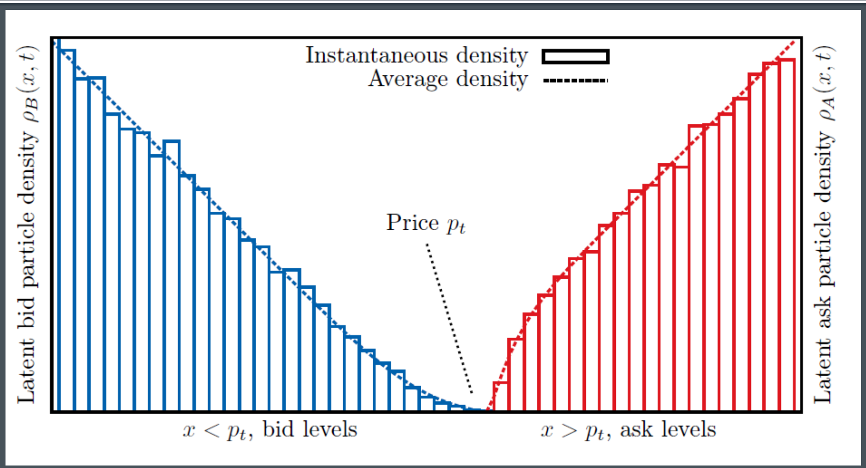
CFM

Some results:

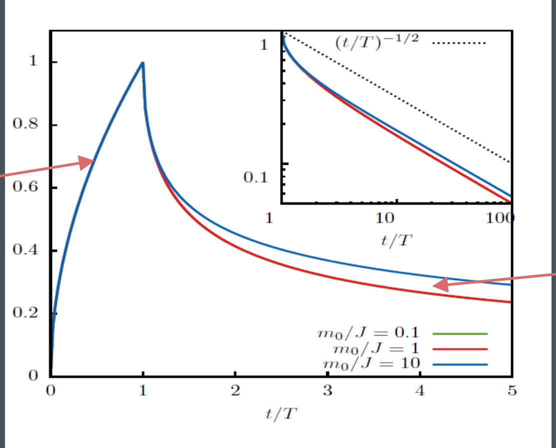
A *non-linear* ‘propagator’ model

$$m_t = m_{t_0} + \sum_{t_0 \le n < t} G(t-n)\varepsilon_n + \sum_{t_0 \le n < t} \xi_n.$$

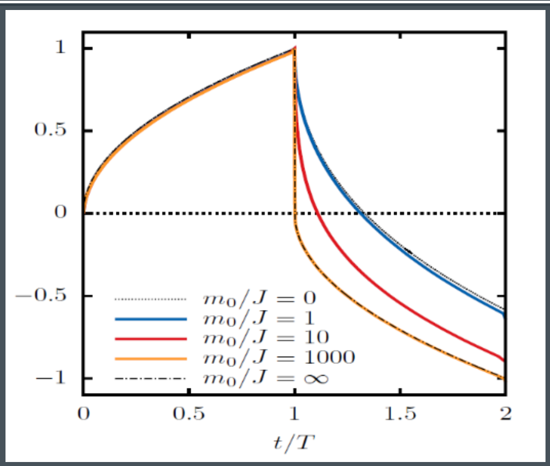
JPB et al. 2004



Square-root

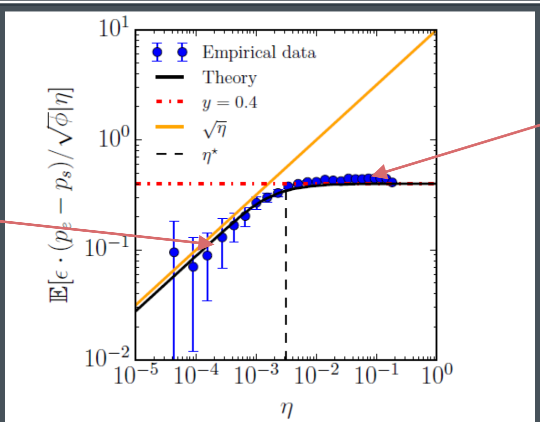


$t^{-1/2}$



Trade reversal: costly!

Linear Q régime



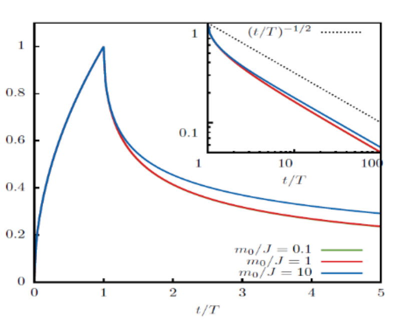
Sqrt regime (indpt of f)

Ancerno

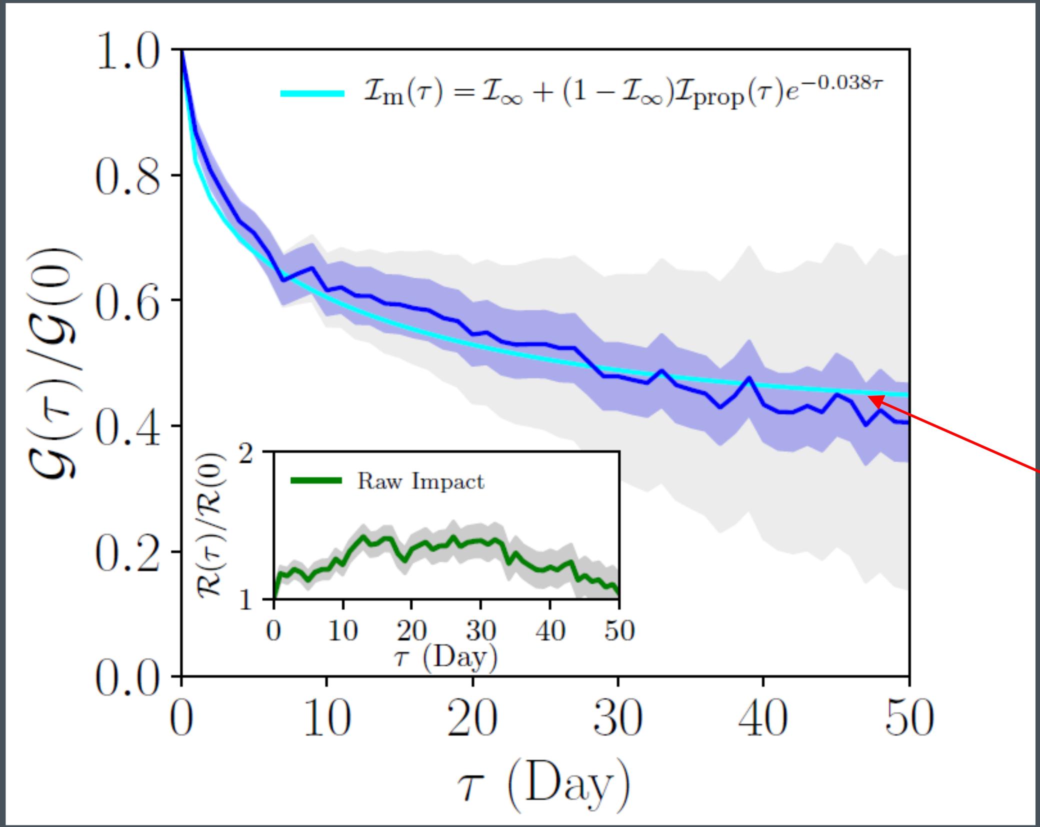
Linear/Sqrt crossover with participation ratio f

What happens when one stops trading?

Impact decay



Slow decay persists over several weeks: worst case scenario for costs

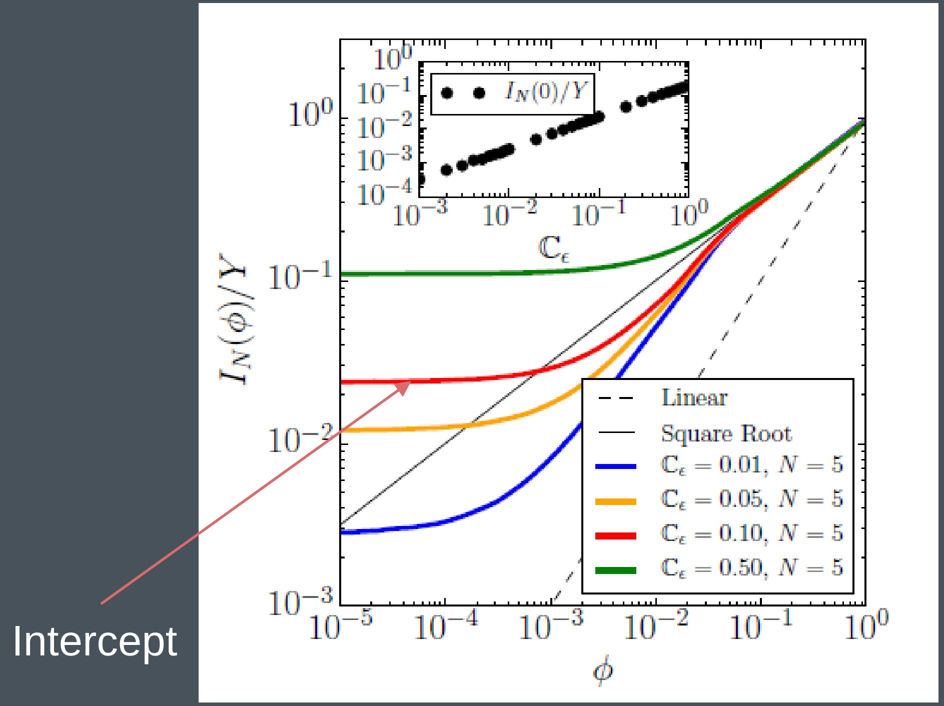


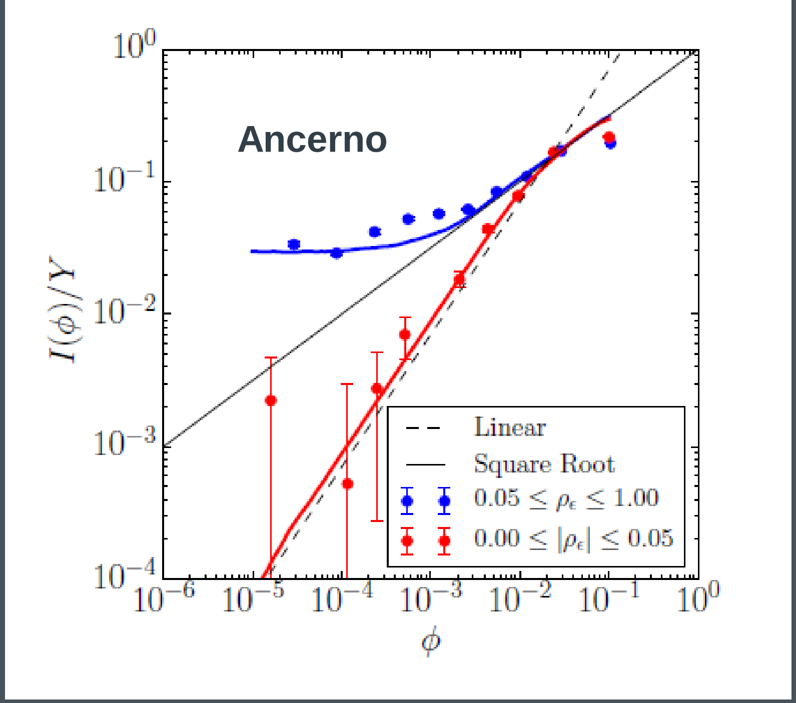
Ancerno data

Permanent impact or « alpha »?

Co-Impact: What happens when others trade simultaneously?

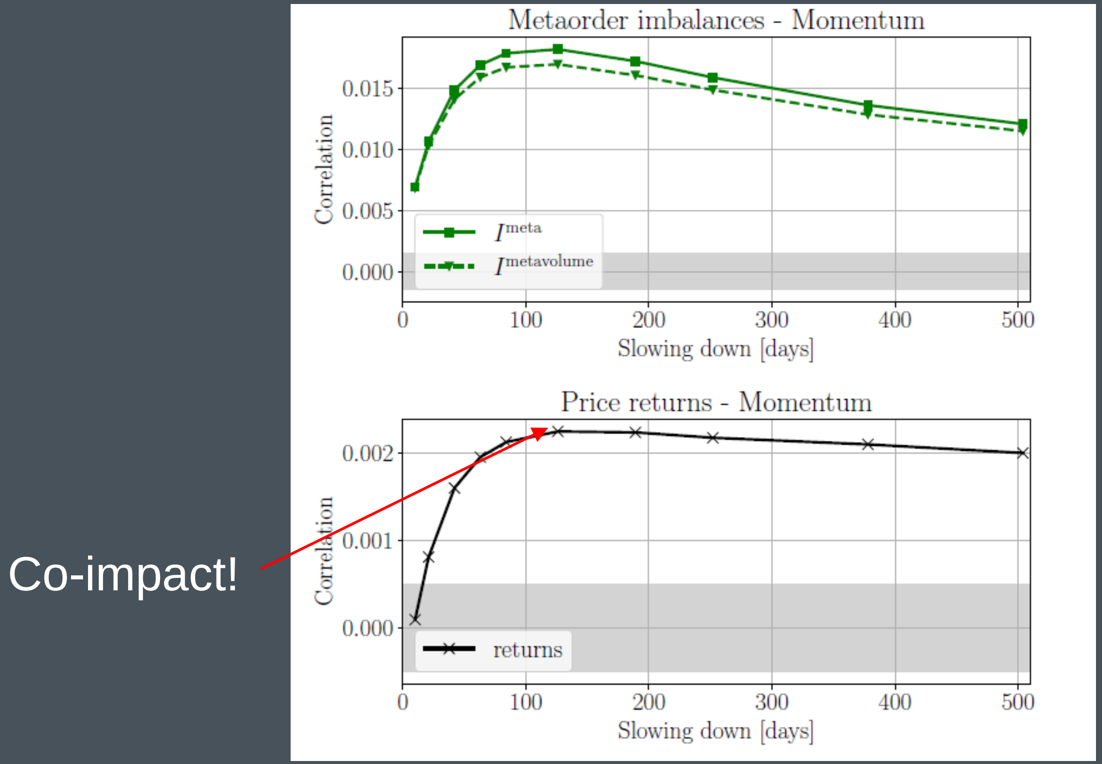
- Data shows that impact is the square-root of the sum, **not** the sum of the square-roots (anonymous trades)
- If others trade in the same direction, an effective intercept appears: a measure of « crowding »

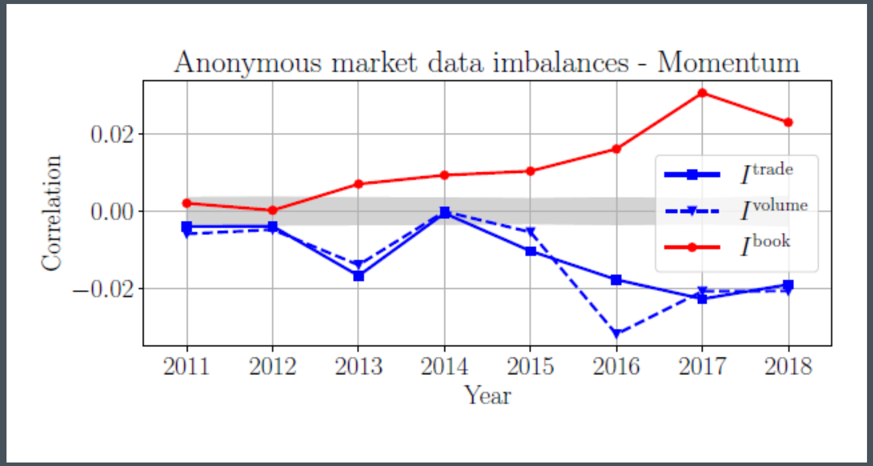




Co-Impact: increases the cost of trading crowded strategies

- Example: Academic (FF) Momentum on stocks → reconstructed slowed down trades
- Co-impact is now eating most of the expected return of FFMOM

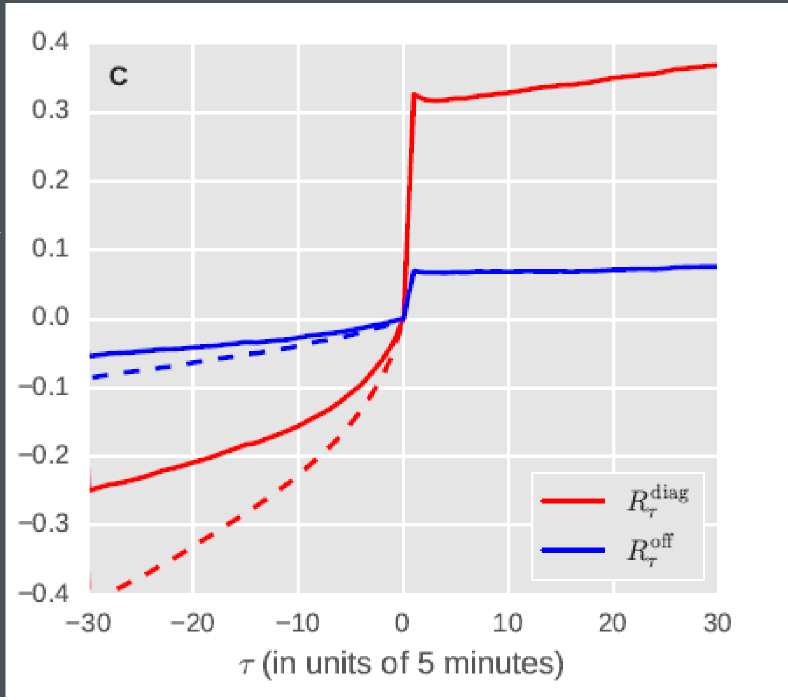




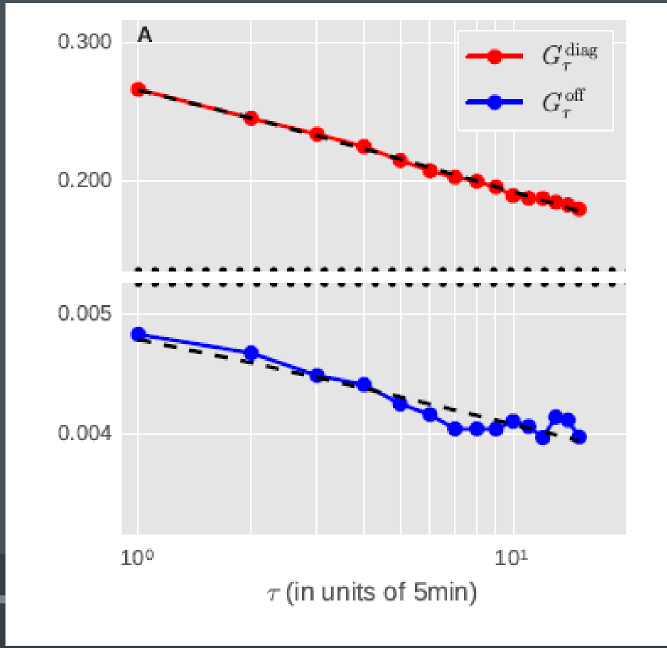
Risk magazine, 2020

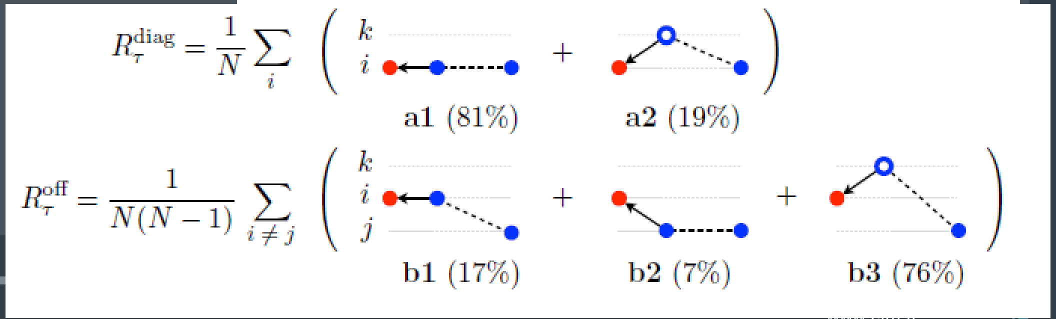
Cross-Impact: what happens to correlated assets when we trade one of them?

- Trading one stock impacts other correlated stocks (weak but significant)
- Relevant for *portfolio trading* (including hedging derivatives)
- Cross-impact must be described by an impact matrix Λ.
- Apparent X-impact mixes « true » X-impact with correlated flows →
- **Apparent X-Impact/ vs. « True »** using public data on US stocks:



Trading 1% ADV with β=1 is 5 x more expensive than with β=0





Cross-Impact: what happens to correlated assets when we trade one of them?

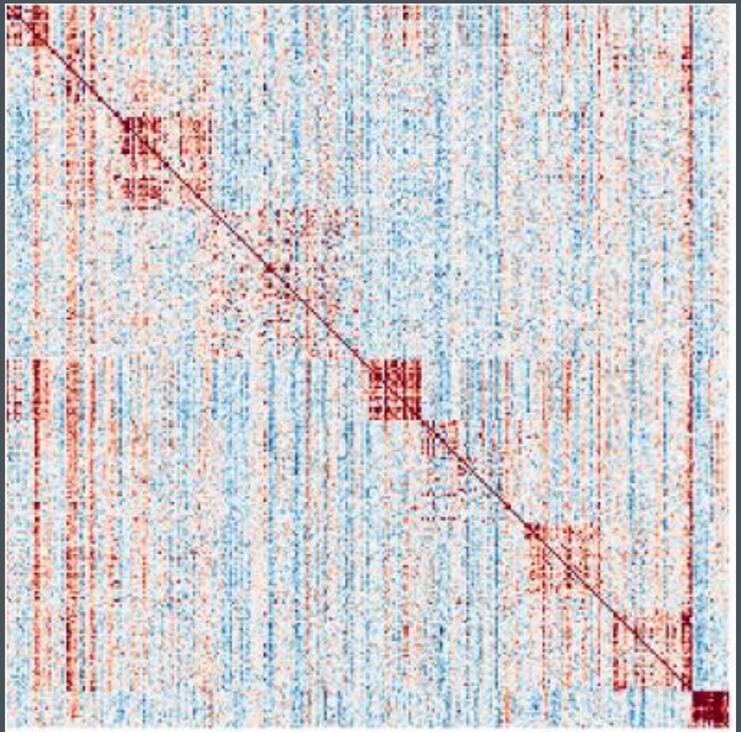
- Cross-impact must be described by an impact matrix Λ, that mixes *return* correlations Σ and *order flow* correlations
- 'Naive' (linear) ML Estimator: $\Delta \mathbf{p} = \Lambda \, \mathbf{Q}$
- For more elaborate cleaning schemes: See

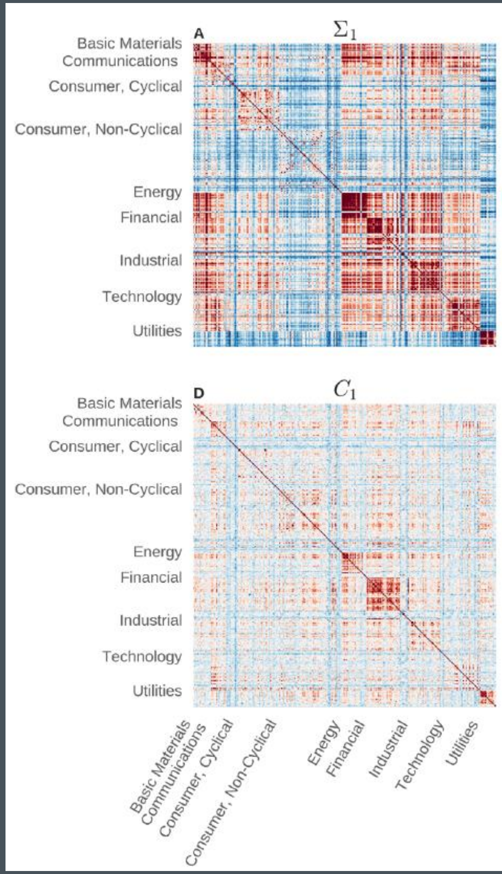
Garcia del Molino, L. C., Mastromatteo, I., Benzaquen, M., & Bouchaud, J. P.
The Multivariate Kyle Model: More is Different.
SIAM Journal on Financial Mathematics, *11*(2), 327-357 (2020)

Tomas, M., Mastromatteo, I., & Benzaquen, M.
How to build a cross-impact model from first principles:
Theoretical requirements and empirical results. *Available at SSRN*. (2020).

- But still no consistent multivariate sqrt model yet!

Λ_{MLE}





Sqrt Impact and Cross-Impact: Intrinsic Market Fragility

Broader Consequences for Market stability/fragility

- Liquidity fluctuations must play a crucial role →

Micro-crises and jumps in prices without news, as seen empirically ever since markets exist

- Volatility-liquidity feedback loop can become unstable → 'flash crashes' (A. Fosset, M. Benzaquen, JPB)
- Cross-impact: Increased synchronisation between markets, in particular in crisis periods (Lillo et al. 2018)

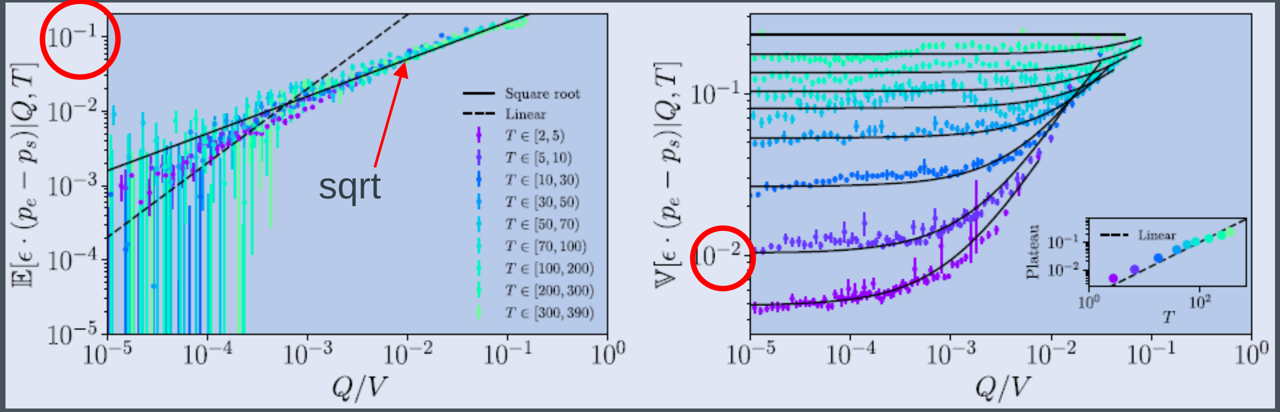
- **Some open questions**:
  - Breakdown of the sqrt law in extreme trading regimes?
  - How does impact really decay?
  - Impact in auctions, dark pools, etc.?
  - Role of multi-time scales in the LLOB framework?
  - How to include LLOB in optimisation algorithms?
  - How to formulate an LLOB theory for X-impact?

(cf. the May 28th 1962 flash crash)

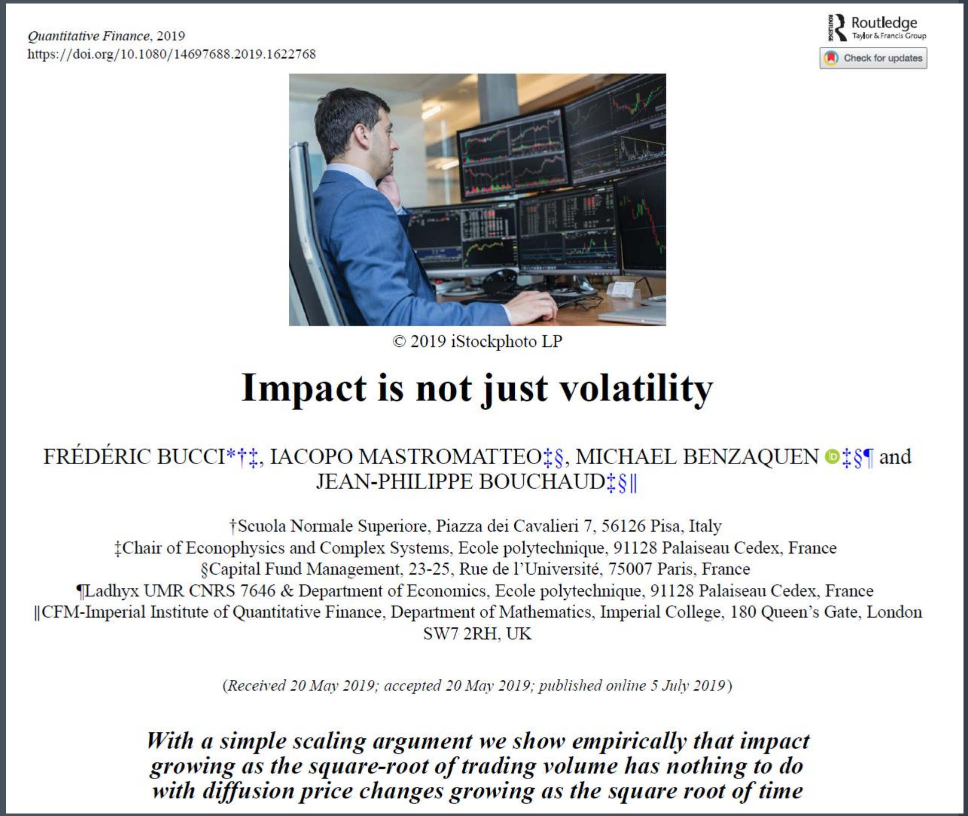
(Flash-) crashes are as old as markets: liquidity is a pyramid sitting on its tip.

The square-root impact law is NOT volatility

- A simple (but wrong) argument: executing Q takes a certain time T ∝ Q
- Price typically moves by $\sigma_1 \sqrt{T}$ → the square root law?
- But a) volatility is unsigned; b) $I(Q) \ll \sigma_1 \sqrt{T}$; c) I(Q) ~ T independent



$$p_e - p_s = \epsilon \cdot \mathcal{I}(Q,T) \times (1 + a\,\eta) + \sigma\sqrt{T}\xi$$


Quantitative Finance, 2019
https://doi.org/10.1080/14697688.2019.1622768

Routledge Taylor & Francis Group
Check for updates

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Impact is not just volatility

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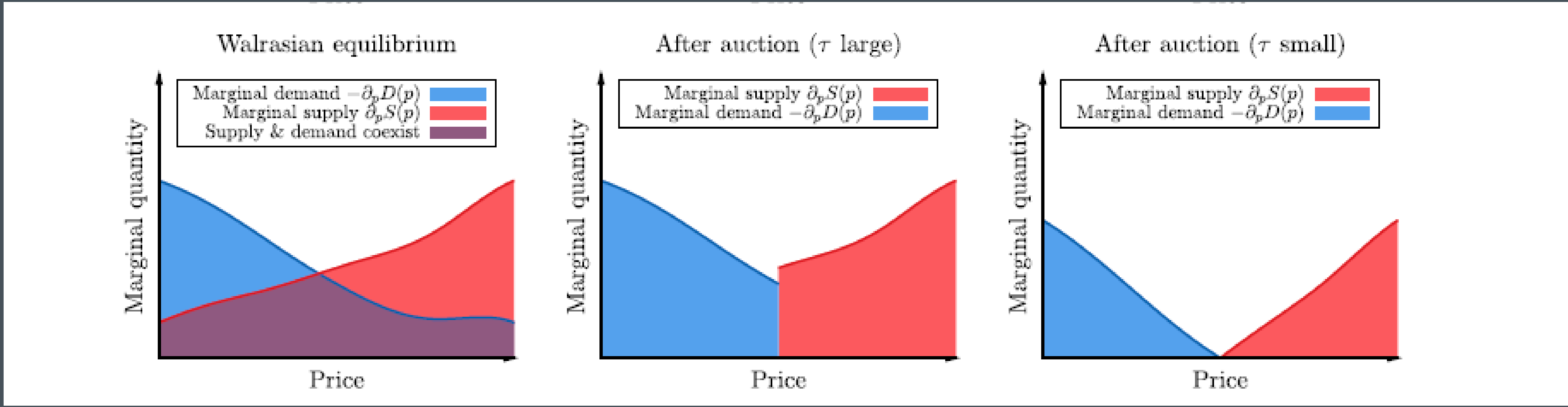
(Received 20 May 2019; accepted 20 May 2019; published online 5 July 2019)

With a simple scaling argument we show empirically that impact growing as the square-root of trading volume has nothing to do with diffusion price changes growing as the square root of time

Sqrt/Linear impact in Auctions



Empirically: 1bp per 1% of auction volume