

# Impact, co-impact, cross-impact

A (short) review

Presented by **Jean-Philippe Bouchaud**

Chairman & Chief Scientist, CFM

**With thanks to** M. Benzaquen, J. Bonart, F. Bucci, J. Donier,  
Z. Eisler, L. Garcia, F. Lillo, I. Mastromatteo, B. Toth

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# Trades, Quotes and Prices

"An impressive book that no serious student of market microstructure can afford to be without. Simultaneously quantitative and highly readable."  
Jim Gatheral, Baruch College, CUNY

"I highly recommend this to anyone who wants to see how physics has benefited economics, or for that matter, to anyone who wants to see a stellar example of a theory grounded in data."  
Doyne Farmer, University of Oxford

"This is a masterful overview of the modern and rapidly developing field of market microstructure, from several of its creators. This book will be an essential resource for practitioners, academics, and regulators alike."  
Robert Almgren, New York University and Quantitative Brokers

The widespread availability of high-quality, high-frequency data has revolutionised the study of financial markets. By describing not only asset prices, but also market participants' actions and interactions, this wealth of information offers a new window into the inner workings of the financial ecosystem. In this original text, the authors discuss empirical facts of financial markets and introduce a wide range of models, from the micro-scale mechanics of individual order arrivals to the emergent, macro-scale issues of market stability. Throughout this journey, data is king. All discussions are firmly rooted in the empirical behaviour of real stocks, and all models are calibrated and evaluated using recent data from NASDAQ. By confronting theory with empirical facts, this book for practitioners, researchers and advanced students provides a fresh, new and often surprising perspective on topics as diverse as optimal trading, price impact, the fragile nature of liquidity, and even the reasons why people trade at all.

**Jean-Philippe Bouchaud** is a pioneer in Econophysics. He co-founded the company Science & Finance in 1994, which merged with Capital Fund Management (CFM) in 2000. He was awarded the CNRS Silver Medal in 1995 and the Risk Quant of the Year Award in 2017.

**Julius Bonart** is a lecturer at University College London, where his research focuses on market microstructure and market design.

**Jonathan Donier** completed a PhD at University Paris 6 with the support of the Capital Fund Management Research Foundation and currently works in the technology sector.

**Martin Gould** currently works in the technology sector. Previously, he was a James S. McDonnell Postdoctoral Fellow in the CFM-Imperial Institute of Quantitative Finance at Imperial College London.

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Bouchaud, Bonart, Donier and Gould  
TRADES, QUOTES AND PRICES

## TRADES, QUOTES AND PRICES

Financial Markets Under the Microscope

Jean-Philippe Bouchaud, Julius Bonart,  
Jonathan Donier and Martin Gould

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# The basics: Impact & Transaction Costs (aka 'slippage')

You have a quantity  $Q \gg V_{\text{ask}}$  to buy – say – what should you expect?

- ▶  $Q$  has to be sliced and diced, and executed over some time  $T$  (~ days)
- ▶ During  $T$ , price will move as  $\Delta p = \pm \sigma\sqrt{T} + \text{Impact}(Q,T)$  [+ « alpha »]
- ▶ The first term is execution **risk**: can be painful but vanishes on average
- ▶ The Impact term is usually **smaller** than  $\sigma\sqrt{T}$  but **always adverse** → « slippage »
- ▶ The alpha term is usually **small** for  $T \sim$  days, but may become significant for fast signals
  
- ▶ **How large are these impact costs** (on top of other costs – fees, spreads, etc.)?
- ▶ How does impact depend on both  $Q$  and  $T$ ?
- ▶ Many more interesting and relevant questions (see below)

# Price impact: orders of magnitude

- ▶ Until the mid-nineties, the lore was that trading a quantity  $Q$  would impact prices as:

$$\Delta p/p \sim Q/M_{cap} \sim 50 \text{ bp for } Q = 100\% V \text{ (} V = \text{Average Daily Volume)}$$

Note: Portfolio Insurance in 1987 should have moved the market by 0.1% (→ underestimates market fragility)

- ▶ Kyle (1985): theory for impact where an insider hides in the flux of noise traders

$$\Delta p/p = \sigma N^{1/2} Q/V \sim 60 \text{ bp for } Q = 1\% V ; \sigma = 2\% ; N=1000 \text{ trades}$$

Note: linear, permanent impact – assumes order flow to be uncorrelated

- ▶ Empirically: the ‘square-root’ law (1995 → 2020)

$$\Delta p/p = Y \sigma (Q/V)^{1/2} \sim 10 \text{ bp for } Q = 1\% V ; \sigma = 2\% ; Y=0.5 \text{ (>> fees)}$$

Note: non-linear, decaying impact – see below

# Sqrt-Impact of Metaorders

A metaorder of size  $Q$  has a sqrt price midpoint impact:

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$

$I(Q)$  is the signed average price change

$Q$  is the volume of the metaorder

$\sigma_T$  is the volatility of the market

$V_T$  is the total volume traded in the market

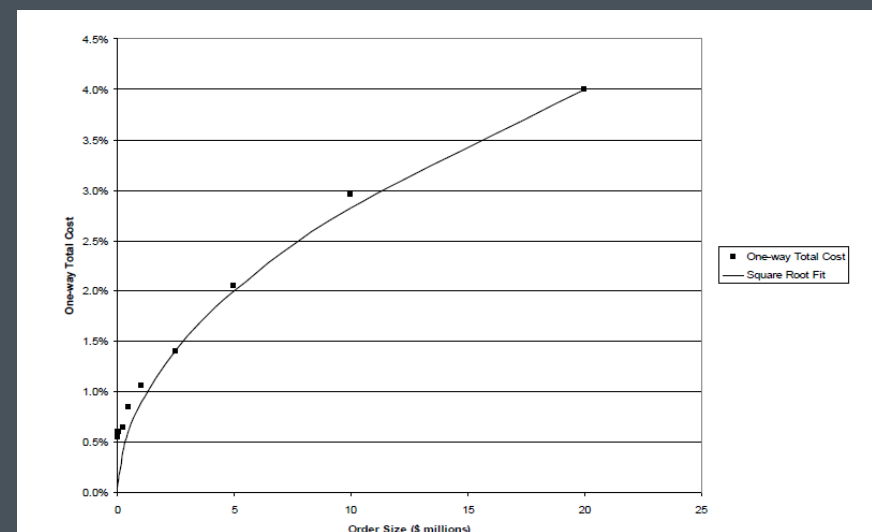
( $Y$  of order 1)

## Important notes:

- ▶ **Impact is usually small compared to vol itself (more later)**
- ▶ Requires a lot of averaging to be seen
- ▶ Beware of (many) conditioning artefacts
- ▶ Note: most data is in a « reasonable » trading regime (neither too big nor too fast, otherwise spreads might considerably increase)

## A universal empirical result?

Independently but consistently reported by many groups since the mid-eighties (Loeb 83 (!), BARRA 95, Almgren 05, Engle, Kissel, JPM, DB, LH, CFM, Ancerno data, AQR)



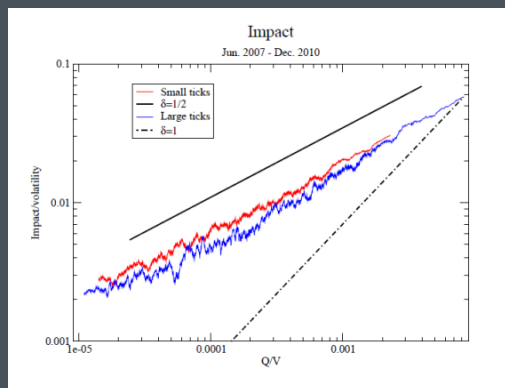
US Stocks, Loeb

# Sqrt-Impact of Metaorders

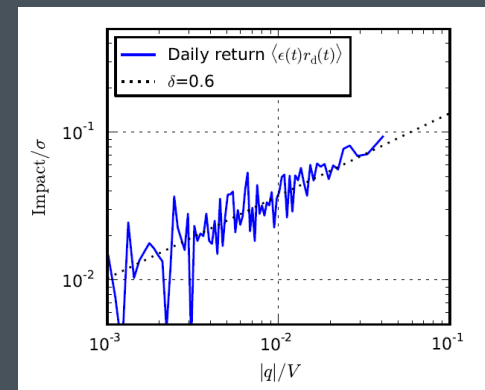
A universal empirical result? (CFM data)

$$I(Q) = Y \sigma_T \sqrt{\frac{Q}{V_T}}$$

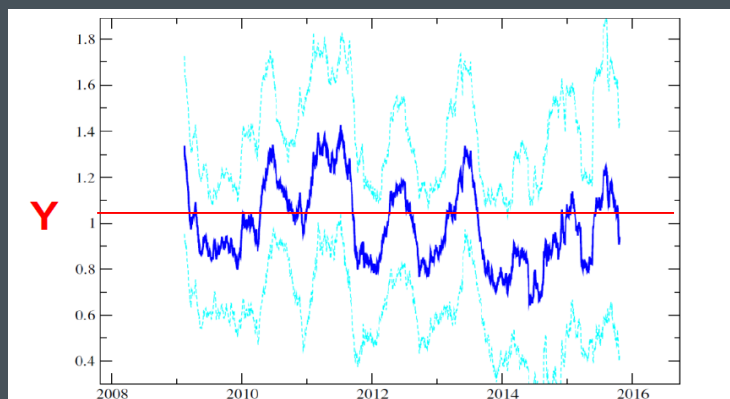
## Futures



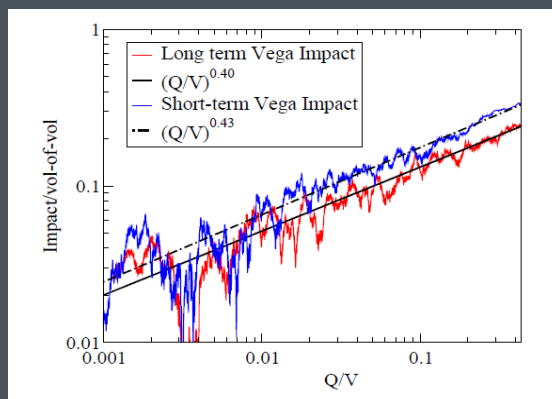
## Intl stocks



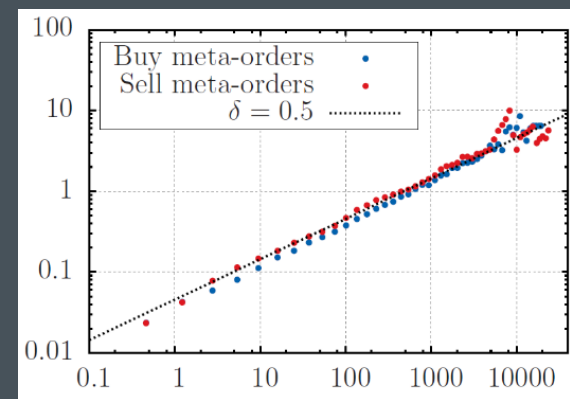
Remarkable stability of  $Y$  (+ noise)



## US stock implied vol



## Bitcoin! (J. Donier)

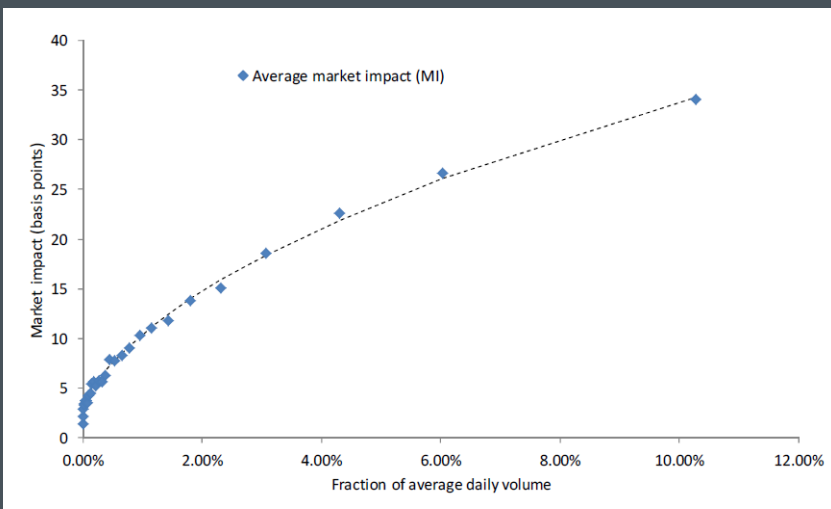


# Sqrt-Impact of Metaorders

A universal empirical result? (AQR, Ancerno)

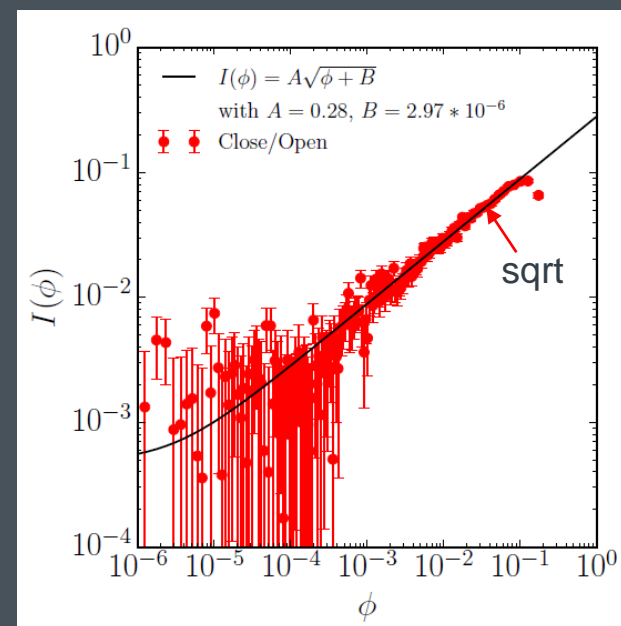
$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$

AQR: Intl stocks



**Note:** execution mostly with *limit* orders  
(Frazzini et al. 2018)

Ancerno: US 2007-2010

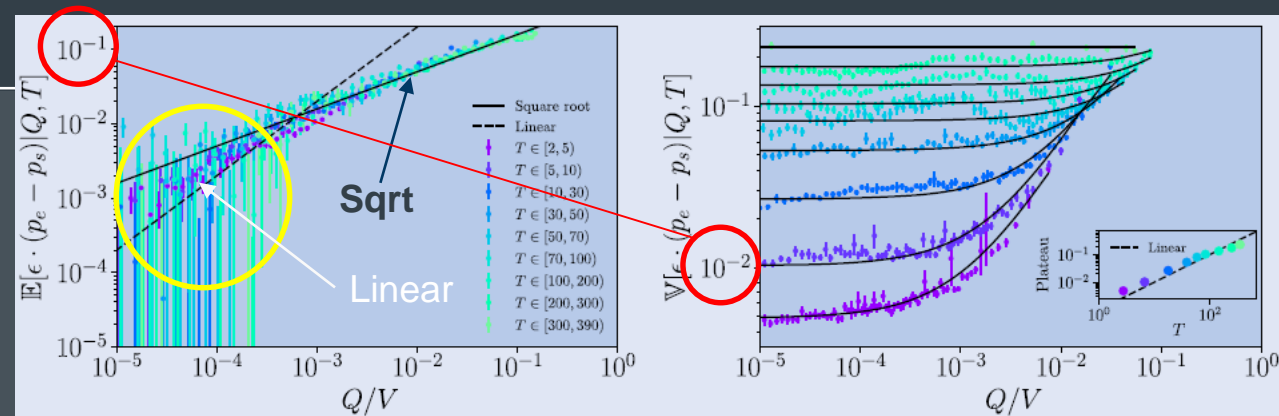


8M metaorders



# The square-root impact law

$$I(Q) = Y\sigma_T \sqrt{\frac{Q}{V_T}}$$



- ▶ Impact is very small compared to volatility
- ▶ Non linear: the second  $Q/2$  impact less than the first  $Q/2$ !
- ▶ Impact is, to first approximation, independent of the execution time of the metaorder – but beware “spread impact”
- ▶ Remarkable stability of results: strategies, markets, execution, period (1980 – 2020), tick sizes, treatment of data,
- ▶ Hints that microstructure and HFT effects are not relevant, only ‘macro-liquidity’ (cf: same with limit orders, w/o HFT)
- ▶ Understanding its origin is important both conceptually and for applications
- ▶ Other relevant questions, e.g. how does this impact decay with time?

# A dynamical theory of *latent* liquidity: the LLOB model (Donier, Bonart, Mastromatteo, JPB)

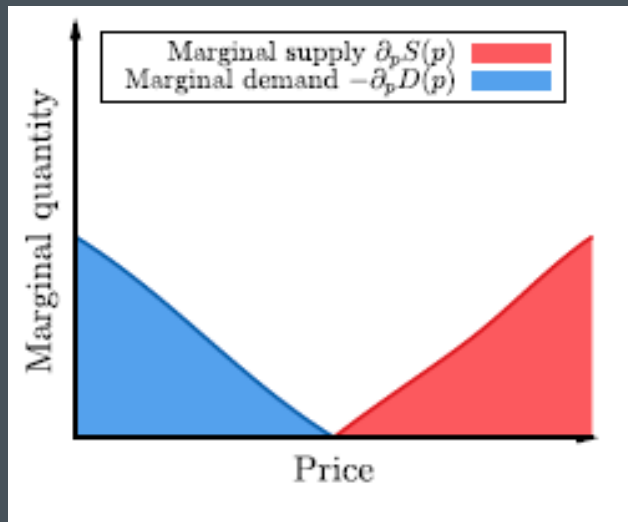
- ▶ A model for the “latent” order book, i.e. macro-liquidity (not micro/HFT liquidity!), inspired from a zero-intelligence ABM

$$\partial_t \varphi_b = D \partial_{xx} \varphi_b - \nu \varphi_b + \lambda \Theta(x_t - x) - R_{ab}(x)$$

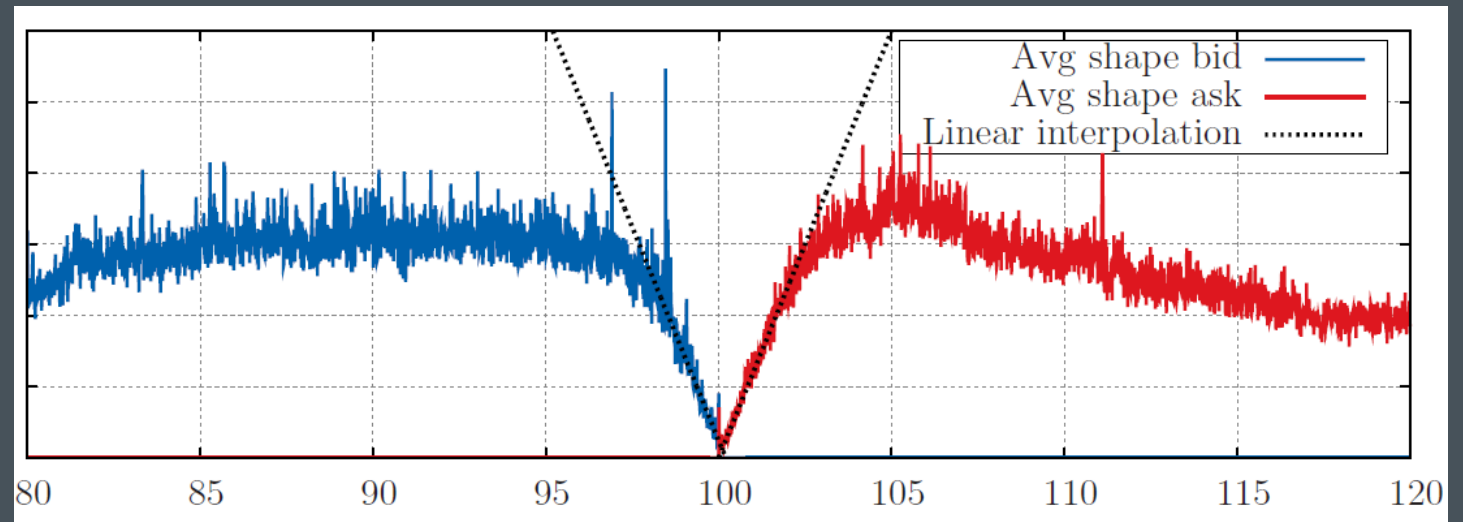
$$\partial_t \varphi_a = D \partial_{xx} \varphi_a - \nu \varphi_a + \lambda \Theta(x - x_t) - R_{ab}(x)$$

$x_t :=$  current price

- ▶ Equilibrium state: a **locally linear** supply/demand curve – liquidity at the current price is at a **V-shaped minimum**



Theory



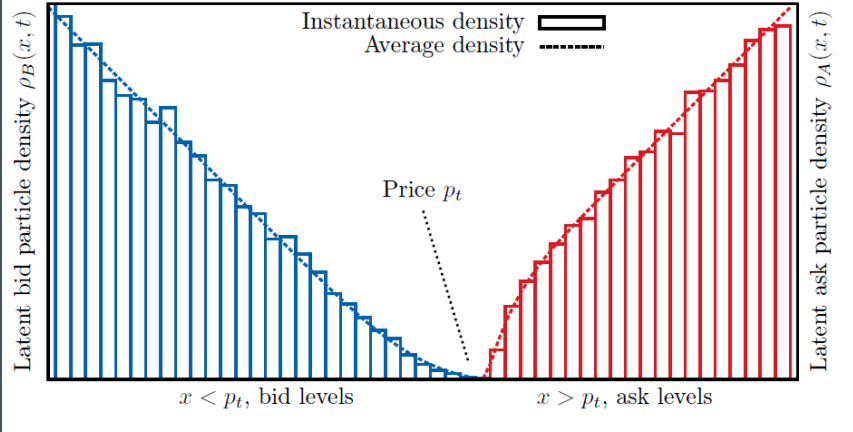
Bitcoin 2012

# Some results:

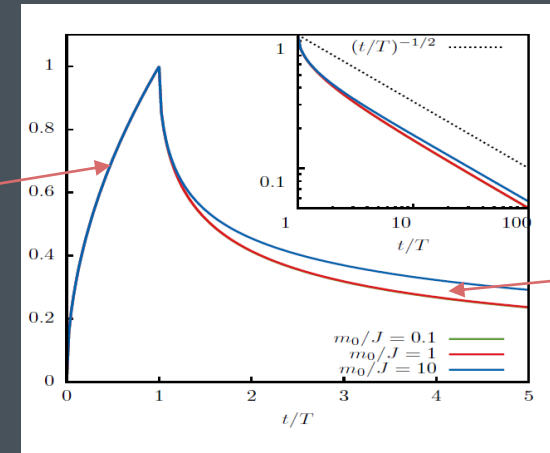
A non-linear 'propagator' model

$$m_t = m_{t_0} + \sum_{t_0 \leq n < t} G(t-n)\varepsilon_n + \sum_{t_0 \leq n < t} \xi_n.$$

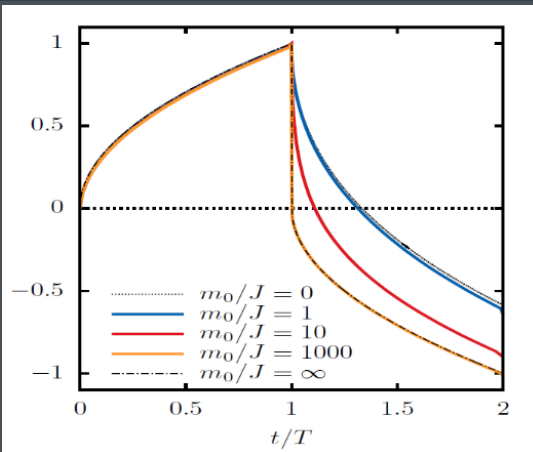
JPB et al. 2004



Square-root

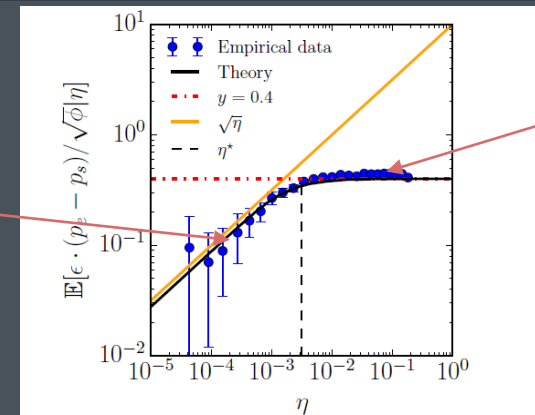


t<sup>-1/2</sup>



Trade reversal: costly!

Linear Q régime



Sqrt regime (indpt of f)

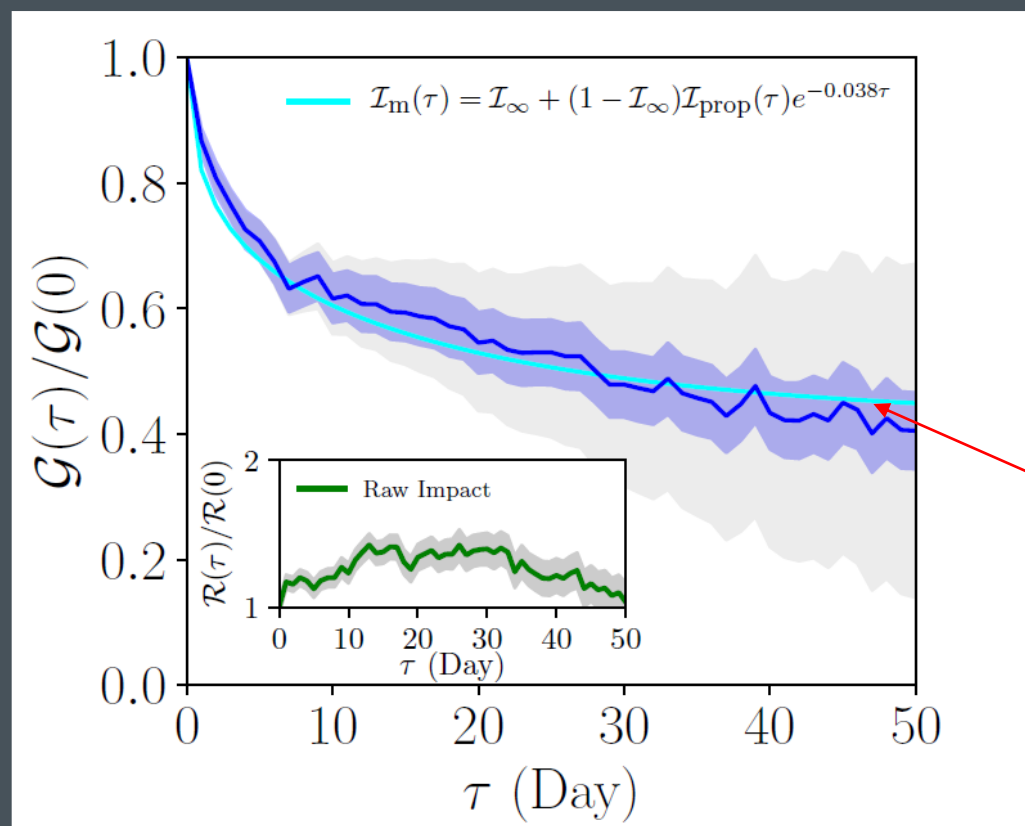
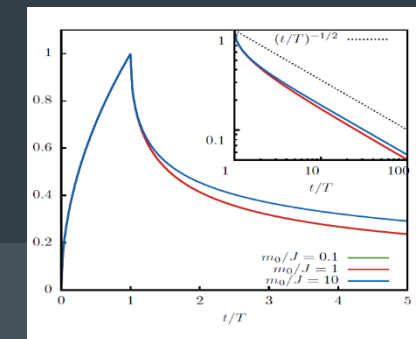
Ancerno

Linear/Sqrt crossover with participation ratio f

# What happens when one stops trading?

## Impact decay

Slow decay persists over several weeks: worst case scenario for costs

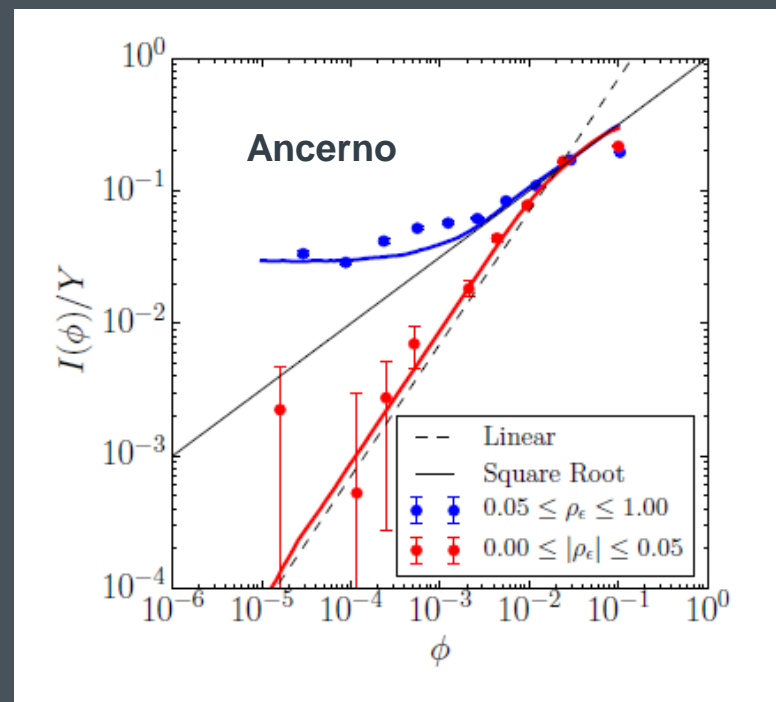
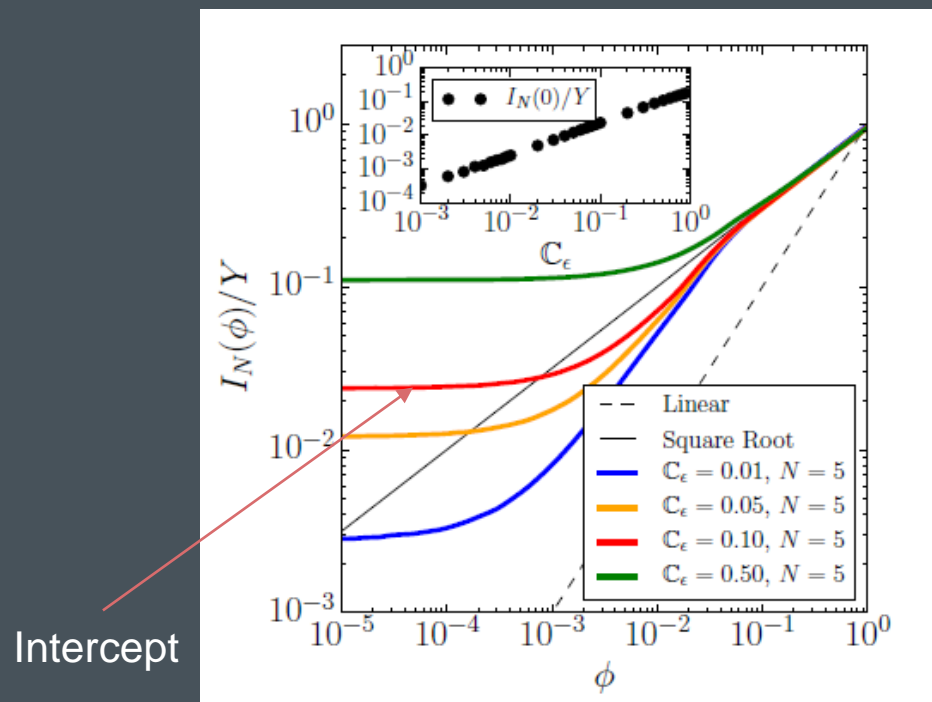


Ancerno data

Permanent impact  
or « alpha »?

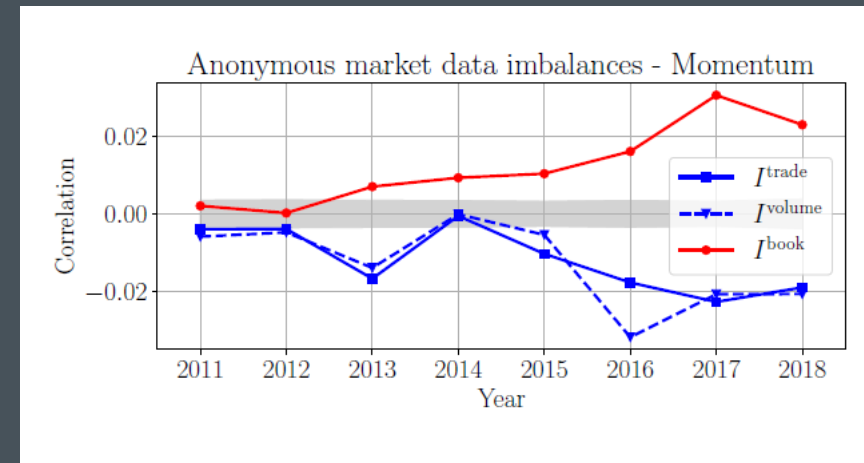
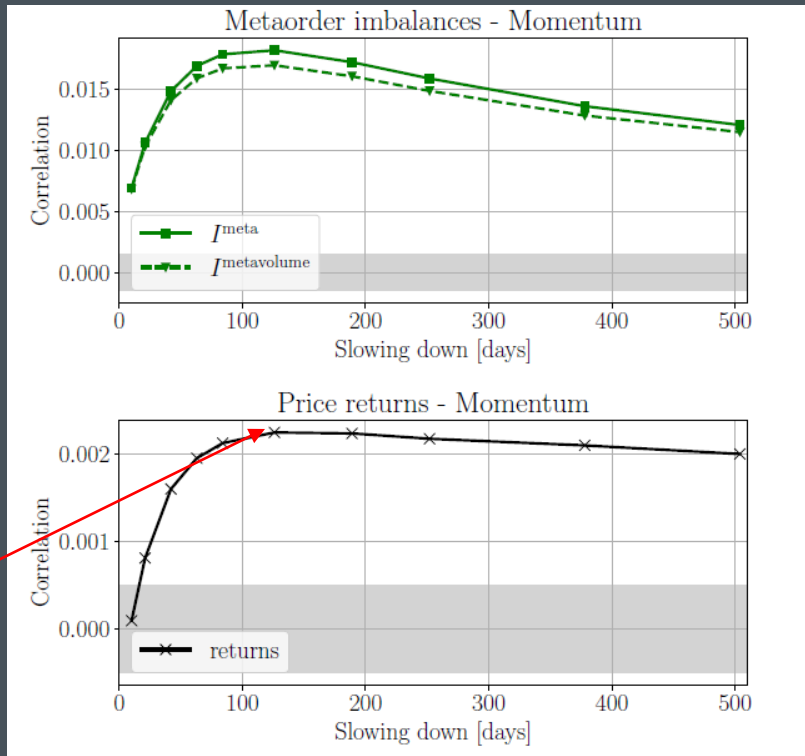
# Co-Impact: What happens when others trade simultaneously?

- ▶ Data shows that impact is the square-root of the sum, **not** the sum of the square-roots (anonymous trades)
- ▶ If others trade in the same direction, an effective intercept appears: a measure of « crowding »



# Co-Impact: increases the cost of trading crowded strategies

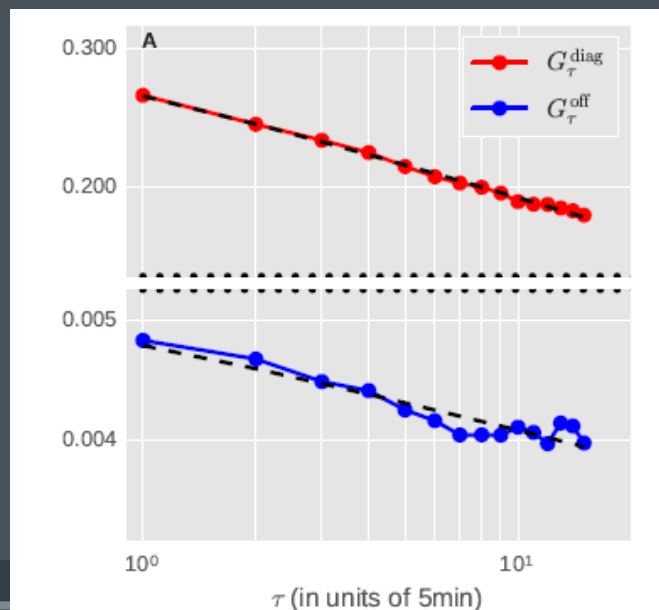
- ▶ Example: Academic (FF) Momentum on stocks → reconstructed slowed down trades
- ▶ Co-impact is now eating most of the expected return of FFMOM



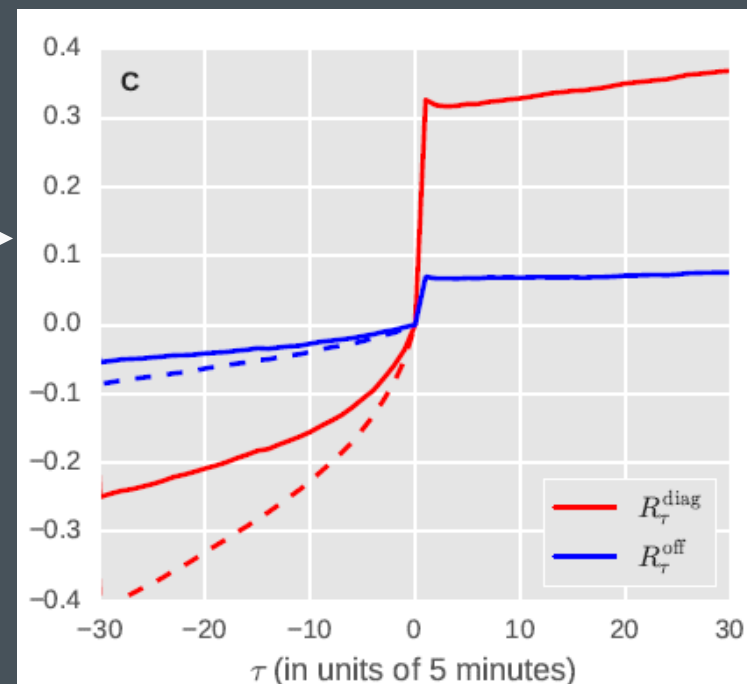
Risk magazine, 2020

# Cross-Impact: what happens to correlated assets when we trade one of them?

- ▶ Trading one stock impacts other correlated stocks (weak but significant)
- ▶ Relevant for *portfolio trading* (including hedging derivatives)
- ▶ Cross-impact must be described by an impact matrix  $\Lambda$ .
- ▶ Apparent X-impact mixes « true » X-impact with correlated flows
- ▶ **Apparent X-Impact/ vs. « True »** using public data on US stocks:



Trading 1% ADV with  $\beta=1$   
is 5 x more expensive than  
with  $\beta=0$



$$R_{\tau}^{\text{diag}} = \frac{1}{N} \sum_i \left( \begin{array}{c} k \\ i \end{array} \left( \begin{array}{c} \bullet \\ \leftarrow \bullet \end{array} + \begin{array}{c} \bullet \\ \leftarrow \bullet \end{array} \right) \right)$$

a1 (81%)      a2 (19%)

$$R_{\tau}^{\text{off}} = \frac{1}{N(N-1)} \sum_{i \neq j} \left( \begin{array}{c} k \\ i \\ j \end{array} \left( \begin{array}{c} \bullet \\ \leftarrow \bullet \\ \leftarrow \bullet \end{array} + \begin{array}{c} \bullet \\ \leftarrow \bullet \end{array} + \begin{array}{c} \bullet \\ \leftarrow \bullet \end{array} \right) \right)$$

b1 (17%)      b2 (7%)      b3 (76%)

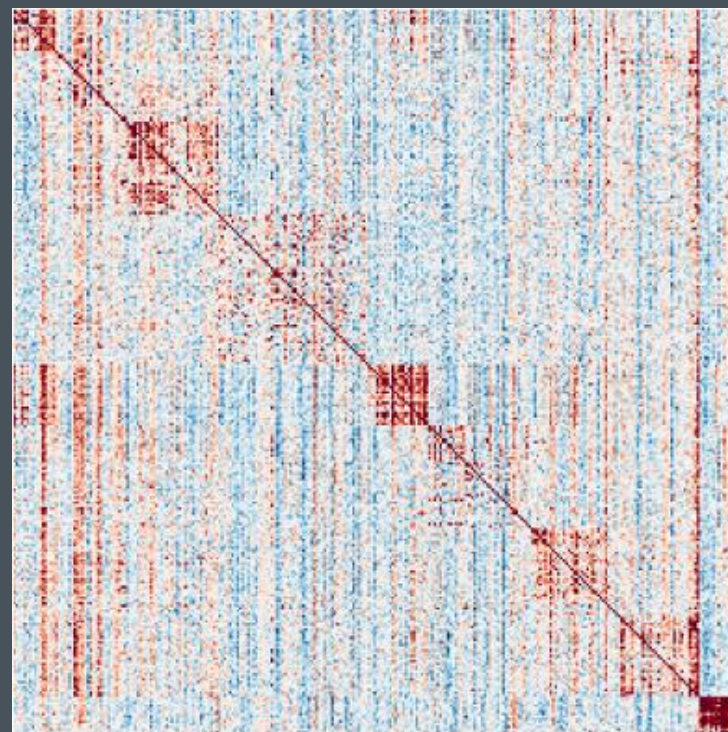
# Cross-Impact: what happens to correlated assets when we trade one of them?

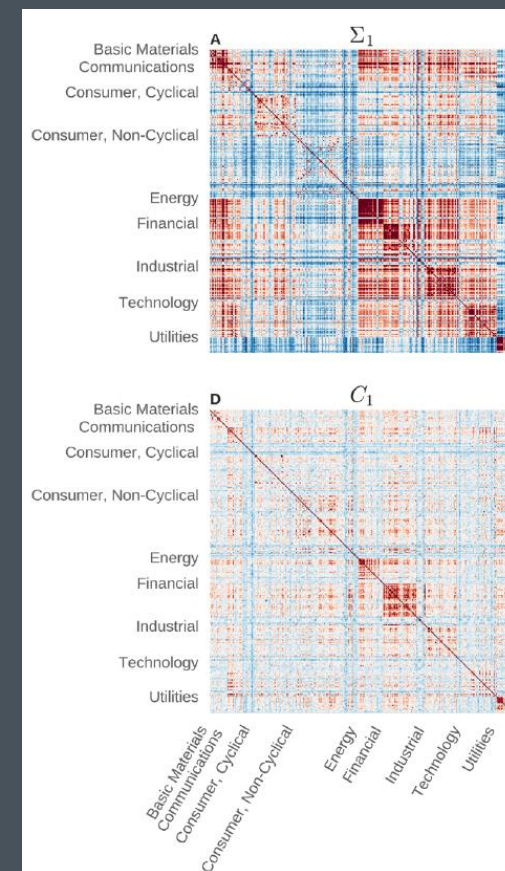
- ▶ Cross-impact must be described by an impact matrix  $\Lambda$ , that mixes *return* correlations  $\Sigma$  and *order flow* correlations
- ▶ ‘Naive’ (linear) ML Estimator:  $\Delta \mathbf{p} = \Lambda \mathbf{Q}$
- ▶ For more elaborate cleaning schemes: See

Garcia del Molino, L. C., Mastromatteo, I., Benzaquen, M., & Bouchaud, J. P.  
 The Multivariate Kyle Model: More is Different.  
*SIAM Journal on Financial Mathematics*, 11(2), 327-357 (2020)

Tomas, M., Mastromatteo, I., & Benzaquen, M.  
 How to build a cross-impact model from first principles:  
 Theoretical requirements and empirical results. *Available at SSRN*. (2020).

- ▶ But still no consistent multivariate sqrt model yet!



$$\Lambda_{MLE}$$




# Sqrt Impact and Cross-Impact: Intrinsic Market Fragility

## Broader Consequences for Market stability/fragility

- ▶ Liquidity fluctuations must play a crucial role → Micro-crises and jumps in prices without news, as seen empirically ever since markets exist
- ▶ Volatility-liquidity feedback loop can become unstable → ‘flash crashes’ (A. Fosset, M. Benzaquen, JPB)
- ▶ Cross-impact: Increased synchronisation between markets, in particular in crisis periods (Lillo et al. 2018)
- ▶ **Some open questions:**
  - > Breakdown of the sqrt law in extreme trading regimes?
  - > How does impact really decay?
  - > Impact in auctions, dark pools, etc.?
  - > Role of multi-time scales in the LLOB framework?
  - > How to include LLOB in optimisation algorithms?
  - > How to formulate an LLOB theory for X-impact?

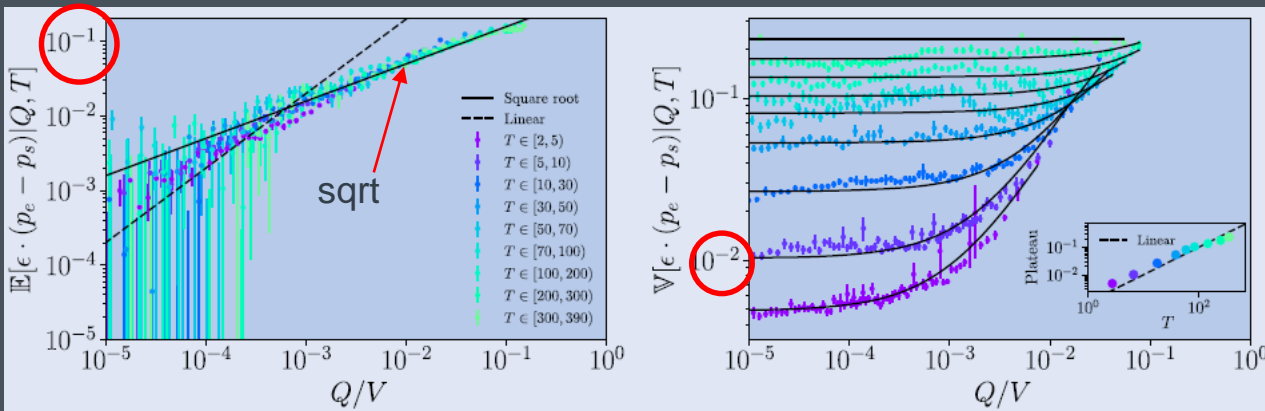


(cf. the May 28th 1962 flash crash)

(Flash-) crashes are as old as markets: liquidity is a pyramid sitting on its tip.

# The square-root impact law is NOT volatility

- ▶ A simple (but wrong) argument: executing  $Q$  takes a certain time  $T \propto Q$
- ▶ Price typically moves by  $\sigma_1 \sqrt{T} \rightarrow$  the square root law?
- ▶ But a) volatility is unsigned; b)  $I(Q) \ll \sigma_1 \sqrt{T}$ ; c)  $I(Q) \sim T$  independent



$$p_e - p_s = \epsilon \cdot I(Q, T) \times (1 + a\eta) + \sigma \sqrt{T} \xi$$

Quantitative Finance, 2019  
<https://doi.org/10.1080/14697688.2019.1622768>



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## Impact is not just volatility

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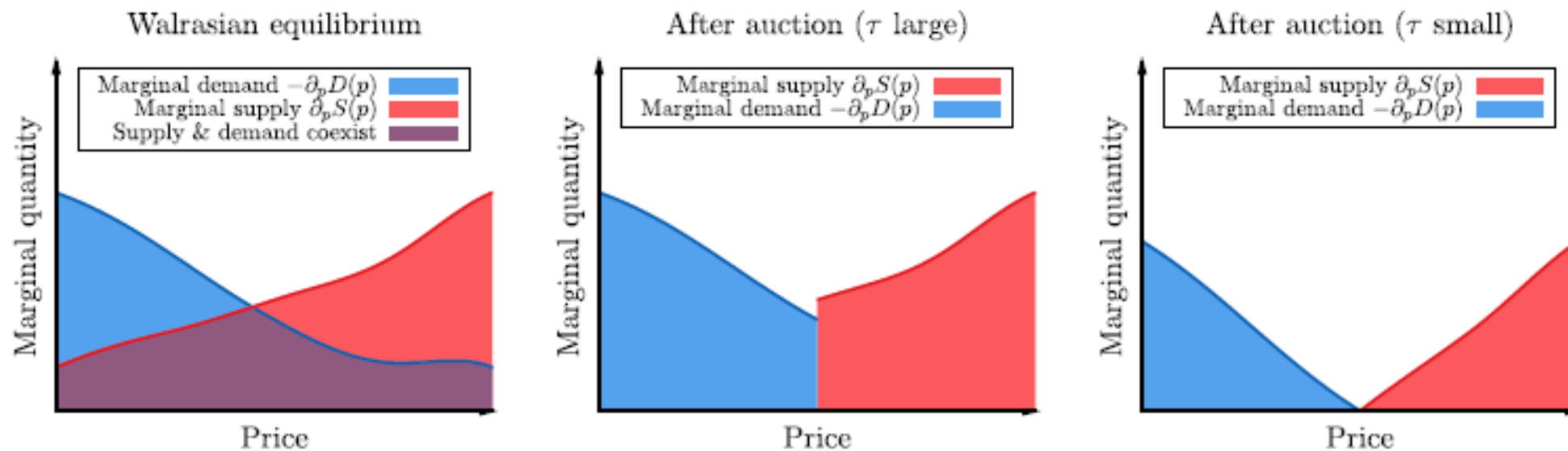
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*With a simple scaling argument we show empirically that impact growing as the square-root of trading volume has nothing to do with diffusion price changes growing as the square root of time*

# Sqrt/Linear impact in Auctions



Empirically: 1bp per 1% of auction volume